

Talks

- **Alexey Bufetov:** *Colored vertex models and random walks on Hecke algebra*

A colored vertex model is an interesting generalization of a six-vertex model. In a stochastic case, it corresponds to particle systems (such as an asymmetric simple exclusion process) with particles of many types. In the talk I will discuss the connection of this model to random walks on Hecke algebra and its several asymptotic applications.

- **Filippo Colomo:** *Arctic curves of the six-vertex model, and the tangent method*

The six-vertex model is known to develop, under suitable choice of boundary condition, and in the scaling limit, phase separation phenomena, that is non-trivial limit shapes and arctic curves.

In this pedagogical talk, we shall discuss the problem of determining the arctic curve of the model. In particular, we shall illustrate the ‘tangent method’, that allows for the determination of the arctic curve in a broad variety of models, and under a wide choice of fixed boundary conditions.

Based on joint works with Andrei Pronko and Andrea Sportiello

- **Ioan Manolescu:** *Rotational invariance in planar FK-percolation*

We prove the asymptotic rotational invariance of the critical FK-percolation model on the square lattice with any cluster-weight between 1 and 4. These models are expected to exhibit conformally invariant scaling limits that depend on the cluster weight, thus covering a continuum of universality classes. The rotation invariance of the scaling limit is a strong indication of the wider conformal invariance, and may indeed serve as a stepping stone to the latter. Our result is obtained via a universality theorem for FK-percolation on certain isoradial lattices. This in turn is proved via the star-triangle (or Yang-Baxter) transformation, which may be used to gradually change the square lattice into any of these isoradial lattices, while preserving certain features of the model. It was previously proved that throughout this transformation, the large-scale geometry of the model is distorted by at most a limited amount. In the present work we argue that the distortion becomes insignificant as the scale increases. This hinges on the interplay between the inhomogeneity of isoradial models and their embeddings, which compensate each other at large scales. As a byproduct, we obtain the asymptotic rotational invariance also for models related to FK-percolation, such as the Potts and six-vertex ones.

Based on joint work with Hugo Duminil-Copin, Karol Kajetan Kozłowski, Dmitry Krachun and Mendes Oulamara.

- **Andrea Sportiello:** *6VM, Fully Packed Loops and the Razumov–Stroganov conjecture*

In this talk we will review (mostly) old and (some) new facts and conjectures revolving around the Razumov–Stroganov correspondence. As well known, this is a family of conjectures, only some of which having been proven, relating two distinct integrable models: on one side the 6VM/FPL with domain-wall boundary conditions, at its combinatorial point, and on the other side the steady-state probability of the $O(1)$ loop model, seen as a stochastic process in the Temperley–Lieb Algebra. We hope to rejuvenate the interest of the listener by recalling how intriguing these correspondences are, and how cute is the proof of the only solved case, namely the one with dihedral symmetry.

Contributed talks

- **Dmitry Chernyak:** *$U_q(\mathfrak{sl}_2)$ -invariant non-compact boundary conditions for the XXZ spin chain*

Using the $U_q(\mathfrak{sl}_2)$ symmetry of the open XXZ spin chain, we introduce new boundary conditions by coupling the bulk Hamiltonian to an infinite-dimensional Verma module on one or both boundaries. We show that for generic values of the parameters the new boundary coupling provides a faithful representation of the blob algebra which is Schur-Weyl dual to the $U_q(\mathfrak{sl}_2)$ action. Modifying the boundary conditions on both the left and the right, we obtain a representation of the (universal) two-boundary Temperley-Lieb algebra. These representations then naturally define various boundary loop models on the lattice, which, thanks to the $U_q(\mathfrak{sl}_2)$ symmetry, can be solved by standard algebraic Bethe ansatz. Moreover, mapping our systems to the (compact) XXZ spin chain with non-diagonal boundary fields we are able to give a purely representation-theoretic interpretation of the so-called Nepomechie constraint. If time permits, we will also explain how to obtain the conformal scaling limit of these models. Based on joint work with A. Gainutdinov, J. Jacobsen and H. Saleur (2207.12772 + unpublished work).

- **Ulrik Hansen:** *Universality of the Critical Random-Cluster Model*

The renormalisation group formalism makes two key predictions about critical planar statistical mechanics models: Their scaling limits should enjoy conformal symmetries and be universal in the sense that they do not depend on the microscopic structure of the lattice on which they are defined. Due to the advent of SLE technology, the first prediction has seen immense progress in the last twenty years. In this talk, we shall focus our attention on the other prediction. More precisely, rotational invariance of a range of critical random-cluster models was established by essentially proving such a universality result across a class of square lattices. In this talk, we discuss how to extend this approach to show universality across a large class of isoradial graphs. This is a joint work with Ioan Manolescu.

- **Mohamed Slim Kammoun:** *Universality for the longest increasing subsequence,*

It is known from the work of Baik, Deift and Johansson that we have Tracy-Widom fluctuations for the longest increasing subsequence of uniform permutations. In this talk, we generalize this result to the Ewens distribution and more generally for a class of random permutations with conjugacy invariant distribution. Moreover, we obtain the convergence of the first components of the associated Young tableaux to the Airy Ensemble.

- **Nikolai Kuchumov:** *A variational principle for multiply-connected domains in domino world*

In this talk we will focus on features of random domino tilings of a multiply-connected domain. We will compare the classical Arctic circle theorem and its analog for the Aztec diamond with a hole, where the height function obtains a monodromy, non-zero increment going around a loop. A more suitable definition of the height function will be presented, which will give us access to a variational principle for multiply-connected domains.

- **Jules Lamers:** *On the structure of the Bethe-ansatz equations of the Heisenberg spin chain*

The Bethe ansatz reduces the spectral problem for the transfer matrix of the six-vertex model with toroidal boundaries, and the periodic spin-1/2 Heisenberg XXZ spin chain, to the problem of solving the Bethe-ansatz equations. Here one often focusses on the ground state and low-lying excitations in the thermodynamic limit. However, even at finite size the Bethe roots exhibit an interesting structure. To probe this structure it is fruitful to study what happens as one varies the parameters of the system. I will discuss two settings in which this can be done. First, the Heisenberg XXZ spin chain. Here we can change the system size L and anisotropy parameter Δ . For the sector with $M = 2$ down-spins I will present some explicit solutions, classify the real and complex Bethe roots, and present a 'critical equation' that determines when Bethe roots switch between real and complex. I will also discuss some results for $M > 2$, and the Ising

limit $\Delta \rightarrow \infty$. Second, the Inozemtsev spin chain. Here $\Delta = 1$ but we can tune the interaction range to interpolate from the nearest-neighbour Heisenberg XXX spin chain to the long-range Haldane–Shastry spin chain, while keeping exact solvability throughout. I will discuss what happens to the Bethe roots as the interaction range increases, and explain how the enhanced (Yangian) spin symmetry in the Haldane–Shastry limit suggests a way to organise the Bethe roots for the Heisenberg XXX spin chain.

The first part is work in progress with J.-S. Caux and R. Koch (U of Amsterdam), generalising results of Essler–Korepin–Schoutens (1992, for $\Delta = 1$) and Imoto–Sato–Deguchi (2019, for $\Delta \geq 1$). The second part is based on joint work with R. Klabbers (Humboldt University zu Berlin).

- **Olivier Marchal:** *Quantization of classical spectral curve and integrable systems*

The purpose of this talk is to review some recent results in quantization of hyper-elliptic spectral curve using topological recursion. Namely, to any algebraic curve $y^2 + P_1(\lambda)y + P_2(\lambda) = 0$ of genus g where P_1 and P_2 are rational functions, we shall associate a formal wave matrix $\Psi(\lambda, \hbar)$ that is solution to a rational Lax pair:

$$\hbar \partial_\lambda \Psi(\lambda, \hbar) = L(\lambda, \hbar) \Psi(\lambda, \hbar), \quad \hbar \partial_t \Psi = A_t(\lambda, \hbar) \Psi(\lambda, \hbar)$$

where t is any isomonodromic time. Moreover, we shall provide an explicit map being spectral times (related to the coefficients of the initial spectral curve) and isomonodromic times as well as an explicit expression for the isomonodromic evolutions and the corresponding Hamiltonians. This talk is based on previous and current work with N. Orantin, M. Alameddine, E. Garcia-Failde and B. Eynard.

- **Maria Matushko:** *Integrable long-range spin chains of Haldane-Shastry type*

Many special properties of the HaldaneShastry long-range spin chain naturally arise from a connection with the spin CalogeroSutherland model. This connection is established by the so-called Polychronakos freezing trick. We show that the Polychronakos freezing trick can be applied to the sets of commuting operators which we call the anisotropic elliptic spin Ruijsenaars-Macdonald operators. It provides the commuting set of Hamiltonians for long-range spin chain constructed by means of the elliptic Baxter-Belavin GL_M R -matrix. We also discuss the trigonometric degenerations based on the XXZ R -matrix.

- **Laurent Menard:** *Spin clusters in random triangulations coupled with Ising model*

We investigate geometric properties of random planar triangulations coupled with an Ising model. This model is known to undergo a combinatorial phase transition at an explicit critical temperature, for which its partition function has a different asymptotic behavior than uniform maps.

In the infinite volume setting, we exhibit a phase transition for the existence of an infinite spin cluster: for critical and supercritical temperatures, the root spin cluster is finite almost surely, while it is infinite with positive probability for subcritical temperatures. Remarkably, we are able to obtain an explicit parametric expression for this probability, which allows us to prove that the percolation critical exponent is $\beta = 1/4$.

We also derive critical exponents for the tail distribution of the perimeter and of the volume of the root spin cluster. In particular, in the whole supercritical temperature regime, these critical exponents are the same as for critical Bernoulli site percolation.

Based on joint works with Marie Albenque and Gilles Scaeffler

- **Mikhail Minin:** *Construction of determinants for the six-vertex model with domain wall boundary conditions*

Determinant representations play an important role in the study of the six-vertex model. In 1982, Korepin listed the properties that fix the partition function of the six-vertex model with domain wall boundary

conditions (DWBC). Then Izergin wrote down the determinant formula satisfying these properties for the trigonometric model. Later, based on Slavnov's determinant for the scalar product, another determinant representation for the partition function of the model with DWBC was obtained in papers by Kostov and by Foda and Wheeler for the rational and a special case of trigonometric parameterization.

We formulate a list of properties of the partition function of the six-vertex model with DWBC for both the rational and trigonometric weights. We provide formulae for the partition function satisfying the properties in terms of a determinant of a matrix, whose elements depend on an arbitrary basis of (Laurent) polynomials in spectral parameters. The resulting expressions include known determinant representations as special cases and provide some more.

The presentation is based on the results of joint work with A. G. Pronko and V. O. Tarasov.

- **Matthew Nicoletti:** *Surface Growth and the Six Vertex Model*

We introduce a family of irreversible growth processes which can be seen as Markov chains on discrete height functions defined on the 2-dimensional square lattice. Each height function corresponds to a configuration of the six vertex model on the infinite square lattice, and irreversibility is due to the fact that the height function has nonzero average drift. The dynamics arise naturally from the Yang–Baxter equation for the six vertex model, namely from a construction called "bijection".

These dynamics preserve the KPZ phase translation invariant Gibbs measures for the stochastic six vertex model, and we compute the current (the average drift) in each KPZ phase pure state. Using this, we derive (and, for general initial conditions, conjecture) the PDE describing the hydrodynamic limit of a non-stationary version of the dynamics acting on quarter plane six vertex configurations.

- **Eric Vernier:** *Duality, Onsager algebra and Ising-type structures in root-of-unity six-vertex models*

The Onsager algebra is an infinite-dimensional Lie algebra which first appeared in Onsager's solution of the two-dimensional Ising model. In [1], we found a surprising connection between the latter and the six-vertex model or its higher-spin generalizations: using Kramers-Wannier duality, a family of N-states integrable vertex models/quantum spin chains were constructed having the Onsager algebra as a symmetry algebra, which were then identified as the six-vertex model and its higher-spin descendants, at specific "root-of-unity" values of the anisotropy parameter. While the integrability of six-vertex models is famously related to an underlying quantum group structure, the enlarged Onsager symmetry could similarly be related to exotic quantum group representations occurring at root of unity. However, certain aspects such as duality remained somewhat hidden in the six-vertex/quantum group formulation. After reviewing the above, in this talk I will revert the logic and show that the (higher spin) root-of-unity six-vertex models can be re-expressed more simply in terms of Ising (clock) spins with products of 2-spins interactions only. The Onsager algebra symmetry emerges naturally in this framework, and the quantum-group related structures and Yang-Baxter equations of the vertex models can be traced back to simpler star-triangle equations in the spin formulation. This is based on [1] E. Vernier, E. O'Brien, P. Fendley, JSTAT (2019) and some work in preparation.